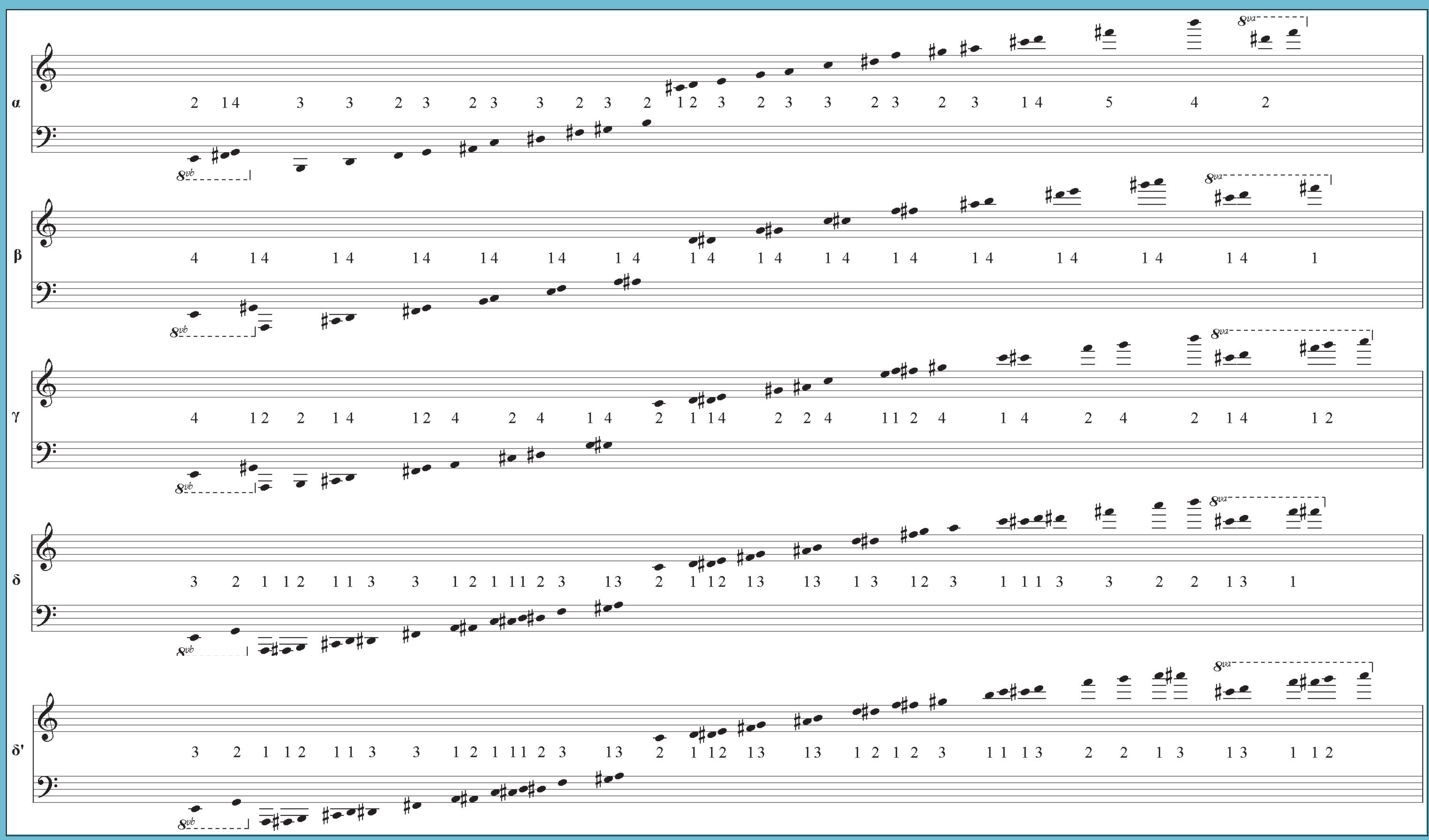
Symmetry and Periodicity in the Sieves of Xenakis's *Shaar*

Dimitris Exarchos

Department of Music, Goldsmiths, University of London, UK

Department of Music and Sound Recording, University of Surrey, UK



1. Sieves in the Sketches of Shaar

Xenakis used sieves γ and δ in Shaar. Sieve δ' is a reconstruction of δ, according to the logic of the identical part of the two sieves (i.e. intervallic pattern evident up to D#5)

Introduction

Many of Xenakis's works of the period following *Jonchaies* (1977) make use of pitch scales (sieves) that share certain characteristics, such as an intervallic content that ranges between 1 and 4 semitones.

In many cases Xenakis applied transformations to his sieves that range from cyclic transposition, which maintains the intervallic structure, to manual alterations such as omitting, adding, or changing one or more elements or segments (thus changing the intervallic structure). In the latter case, analysis can reveal properties common to different versions and thus enable results concerning the construction of sieves.

This poster presents an analysis of the sieves of *Shaar* (1982, for string orchestra). The treatment of these sieves in the pre-compositional sketches provides insight to our understanding of Xenakis's approach to sieve-construction.

The Sieves of Shaar

In the sketches of Shaar Xenakis experimented with four different sieves, labelled α, β, γ and δ, from which he used sieves γ and δ. All sieves are shown in figure 1; the numbers attached to each pitch denote the interval between that pitch and the next (in semitones).

Sieve α bears an irregular, non-repetitive structure, but the three that follow are periodic with periods 5 for β and 7 for γ and δ.

Xenakis commented on the page of the sketches: "Sieves based on a period, difficult for 12 (5, or 7)". Sieve β is a perfect example of interlocking 4ths. When two 4ths interlock the periodicity is the 4th itself (with intervallic structure: 1 4 and so on). But after 12 reprises of the 5-semitone period the sieve starts repeating in terms of octave-equivalent pitches. This would give a greater period of 5·12 = 60 semitones. The case of sieves γ and δ is not as straightforward. Sieve δ' is a reconstruction of δ, according to the intervallic pattern evident in the identical part of the two sieves, i.e. up to D#5 (where C4 is the middle C).

Both γ and δ' are based on the period of the perfect 5th. In both these two sieves, the octave-equivalent pitches would appear at point 7·12 = 84 semitones. But the period of the 5th is not apparent, because the intervals that this period is broken down to, do not always appear in the same order. In sieve γ each cycle of 5th consists of a different permutation of three intervals that the 5th is broken down to: 1, 2, and 4; and in sieve δ' of permutations of the four intervals 1, 1, 2, and 3. The sketches provide a simple way of determining the order the permutations appearing in sieve δ up to D#5 (see figure 1); sieve δ' is precisely the reconstruction of δ according to this system of intervallic permutations.

The number of all the possible permutations of 1, 1, 2, and 3 is 12. As figure 2 shows, Xenakis wrote down in a column the six permutations that correspond to all possible positions of the interval of the semitone, and with 3 always preceding 2; in a second column to the right, he wrote the remaining six permutations where 2 precedes 3. There is therefore a symmetric relation between the two columns. He started on the top left entry: 3 2 1 1. These are the first four intervals in sieve δ'. He then used the permutation on the bottom right: 2 1 1 3. The connecting lines in the figure show the order of appearance of these permutations. The process continues similarly: The second permutation of the left column is followed by the second to the last permutation in the right column, and so on. The last permutation marks the end of the period of the sieve; the first permutation would appear again at point 84, and the sieve would be reproduced at the octave equivalent after 7 octaves. We see that the intervallic structure of sieve δ is identical to that of δ' but the former has an interval of 3 instead of 2 semitones (D#5-F#5 in δ instead of D#5-F5 in δ') and an additional minor 3rd (F#6-A6). Xenakis used, peculiarly, a sieve that is based on the system of intervallic permutations noted in the sketches, but with these two deviations.

The sketches do not provide the system for the permutations of sieve γ, but we can reconstruct it. The possible permutations of three distinct elements are 6. In figure 3, the left column shows five of these permutations, whereas at its bottom the initial permutation (4 1 2) re-appears. In the second column each permutation is the "retrograde" of the corresponding one in the left column. Again, in the right column there are only five distinct permutations, with the top and bottom entries being the same. The order of appearance of each permutation in the sieve is the same as that of sieve δ'. The reason for having only five distinct permutations in each column is related precisely to their order of appearance. If the bottom left entry had the 6th remaining permutation (that would be 2 1 4), its retrograde at the bottom of the right column would be the same as the initial one (4 1 2); thus the two first permutations in the intervallic structure of the sieve would be identical. In order to avoid this, we enter the retrograde of the remaining permutation instead (which is equivalent to swapping the two permutations at the bottom of the two columns). The consequence of this is that the initial permutation (4 1 2) appears again as the 11th permutation, before the sieve reaches its period (i.e. it appears at point 70 instead of 84). Thus, the three final intervals in sieve γ (4 1 2) do not denote the recurrence of the period; this would actually happen at point 84, after all the 12 permutations of the table in the bottom figure have appeared.

3	2	1	1	2	3	1	1
3	1	2	1	2	1	3	1
1	3	2	1	1	2	3	1
1	3	1	2	1	2	1	3
1	1	3	2	1	1	2	3
3	1	1	2	2	1	1	3

2. Intervallic permutations in sieve δ of Shaar

4	1	2	2	1	4
1	2	4	4	2	1
4	2	1	1	2	4
2	4	1	1	4	2
1	4	2	2	4	1
4	1	2	2	1	4

3. Intervallic permutations in sieve γ of Shaar

Outside-Time Structures and Metabolae

The permutations of intervals in the process of constructing the sieves of *Shaar* relate to the notion of the Xenakian *metabola*. This notion refers to the possible transformations of sieves. For Xenakis, sieves are the primary example of outside-time structures; that is, structures whose elements can be arranged hierarchically (from the lower to the higher, or from the smaller to the larger), as opposed to inside-time structures, where elements are sequenced in such a way that our discourse would necessitate the notions of "before" and "after" in order to describe them. In short, metabolae refer to the transformations applied to outside-time structures in order to produce new ones (still in the outside-time category). It follows from this that transformations that might produce new structures, are referred to as metabolae only if they are applied to an existing outside-time structure. In practical terms, these metabolae produce new versions of the outside-time structures they are applied to, and offer the possibility of constructing the sketch of the whole or part of a composition. Metabolae are in this way situated between the outside- and the inside-time.

Conclusion

In general, permutations of time-intervals do produce different structures. In the sieves of *Shaar* we do not see a clear case of metabola; however, we can see such a process in the micro-structural level, in the different segments of a perfect fifth in sieves γ and δ (e.g. each segment in figures 2 and 3). Several permutations of three or four intervals, respectively, are placed in a certain succession; the result is a chain of permutations that renders the intervallic structure of the sieve. Therefore, the term "metabola" would not be used for these sieves of Shaar in the same way as, for example, in the sieves of *Nomos Alpha* (where Xenakis constructed a "departure function" and produced several sieves by altering the initial formula according to predetermined steps); here we have a case of transformations of the basic set of intervals that make up the intervallic structure of the sieve. This process is significant precisely because it shows that metabolae are processes that can demonstrate most clearly the complex relation between the outside- and the inside-time: it is a clear case where Xenakis uses an inside-time organising principle (succession of permutations) to construct an outside-time structure.

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